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DRIFTING IN A VERTICAL ATTITUDE**

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Aero-Astronautics Department
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WAVE-INDUCED MOTIONS OF A LARGE ROCKET VEHICLE DRIFTING IN A VERTICAL ATTITUDE

BY J. J. Leendertse

ABSTRACT

Results are presented of a theoretical and experimental laboratory study of the movements of a large solid-propellant rocket vehicle drifting in a vertical attitude in uniform waves and in the wave environment of the open sea. Measurements were made of the movement in heave and pitch of a 1-120 scale model of a 175-ft long vehicle. Experimentally obtained results compared well with those obtained analytically.

It appears that heave and pitch are linear functions of the wave height and nonlinear functions of the wave frequency. Pitch and surge are coupled.

On basis of the linear response of the vehicle to uniform waves, statistical features of the response in heave and pitch to ocean waves with a particular spectrum are calculated.

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I. INTRODUCTION

The object of this study was to determine the motions of a large rocket vehicle drifting vertically in the wave environment of the open sea prior to a sea launch. The waves in this environment (hereinafter called the "seaway") vary considerably in height, shape, and period, depending on the wind, the water depth, and the wind-affected sea area or fetch.

At present the best representation of the seaway can be made by use of a mathematical model of a random process, which presents no values of the environment as a function of time, but presents statistical values such as averages and the frequency of occurrence of a certain water level.⁽¹⁻⁴⁾ In this approach the seaway is taken as the sum of a large number of uniform wave trains, each with different amplitudes, and the profiles of the individual wave trains are assumed to be sine curves according to Airy's theory.⁽⁵⁾

The magnitude of the amplitudes of the wave trains and their distribution over a range of frequencies can be calculated by a standard method from the wind velocity, wind duration, and the fetch.⁽³⁾ The result of such a calculation can be presented in the form of a wave spectrum, which is a distribution of the mean squares of wave amplitudes of the seaway in a given increment of the frequency over the wave frequencies. Wave spectra in the open sea can also be obtained directly from records of floating or shipborne wave recorders.^(6,7,8)

The study of the motions of the vehicle in a seaway described by a particular spectrum was made in two parts. In the first part the responses

of the vehicle to uniform waves with different heights and frequencies were studied theoretically and experimentally. For the experimental work, the movements of 1-120 scale models of the vehicle were measured simultaneously with the wave in a wave tank (Fig. 1).

In the second part of the study, the responses of the vehicle to seaways, the latter represented by mathematical models of the random process of the ocean surface elevation, were studied, and statistical features of the responses to a particular seaway were obtained.

II. THEORY

THEORY OF MOVEMENTS INDUCED BY UNIFORM WAVES

General

The movements of a rocket vehicle drifting in a wave environment can be described by considering the movements of its mass center relative to a rectangular coordinate system. In analogy to the movements of a ship in a seaway, the movements of the vehicle in its six degrees of freedom are taken as follows (Fig. 2):

heave = vertical motion of the mass center

surge = horizontal motion of the mass center in the direction of the wave propagation (drift)

sway = horizontal motion of the mass center normal to the direction of the wave propagation

pitch = angular motion in a vertical plane through the direction of the wave propagation

roll = angular motion in a vertical plane normal to the direction of the wave propagation

yaw = angular motion around the vertical axis

Since the vehicle studied is symmetrical in any horizontal cross section, it may be assumed that coupling exists only between the wave train and the modes of movement in the vertical plane through the direction of wave propagation: namely, heave, surge, and pitch.

For the purpose of analysis the vehicle is approximated by a series of cylinders of different dimensions, as shown in Fig. 3, and is assumed to be drifting with zero speed in a uniform wave train.

The wave profile is approximated by a sine curve, and the water particles are assumed to describe circular or elliptical orbits, depending on the wavelength-depth ratios. (9)

The origin of the coordinate system that will be used to describe the waves and the movements of the vehicle is taken at the still-water level in the axis of the series of cylinders. The X axis is taken positive in the direction of the wave propagation, and the Z axis is vertical, and positive upward.

The horizontal and vertical displacements of a particle from its mean position located at a distance z under the still-water surface are

$$\xi_x = A \frac{\cosh k(d+z)}{\sinh kd} \sin(kx - \omega t - \epsilon) \quad (1)$$

$$\zeta_z = A \frac{\sinh k(d+z)}{\sinh kd} \cos(kx - \omega t - \epsilon) \quad (2)$$

where

ξ_x = horizontal displacement of a particle

ζ_z = vertical displacement of a particle

A = amplitude of the wave, which is equal to half the wave height H

k = $2\pi/\lambda$

λ = wave length

ω = circular frequency ($= 2\pi/T$)

ϵ = phase angle

t = time

T = wave period

d = water depth

Heave

Considered initially in this analysis are the movements of one of the cylinders of the vehicle. It is assumed that the presence of the sections of the vehicle in the fluids does not change the original pressure distribution in the wave, according to the so-called Froude-Krylov hypothesis.⁽¹⁰⁾

If Z is the upward displacement of the mass center of the section, the equation of motion can be written in the manner presented by Wilson¹¹ for the movement of ships in regular waves as

$$F_{z_n}(p) + S = M_n \ddot{Z} + M''_{z_n} (\ddot{Z} - \dot{w}_n) + 1/2 \cdot C_d F_n (w_n - \dot{Z})^2 + M_n g \quad (3)$$

where, for the particular section

$F_{z_n}(p)$ = total vertical upward force from the wave pressures on the horizontal planes of the section

S = total connection force with other cylinders

M_n = mass of the section

M''_{z_n} = added mass in heaving motion (nth plane)

C_d = drag coefficient

F_n = significant cross section of the section

ρ = density of water

g = acceleration due to gravity

w_n = average of the vertical velocity of the water on the nth plane

\dot{w}_n = average of the vertical acceleration of the water on the nth plane

The first term on the right side of Eq. (3) presents the inertia force due to the section itself; the second term summarizes the influence of the surrounding water. The term M''_{z_n} presents the so-called added mass,^(12,13) associated with the transition of the nth section to the n + 1th section.

The third term is a damping term. The damping is due to the frictional resistance of the wetted surface, the generation of eddies, and the generation of waves with the vehicle center as their origin.^(14,15) Generally the damping force is a quadratic function of the velocity, but following the method used for investigation of ship motions,⁽¹⁰⁾ a linearization is used. The effect of the damping as a whole will be discussed later.

The vertical velocity \dot{w}_n and the vertical acceleration \ddot{w}_n are obtained by averaging the particle velocities in the considered planes at the transition of the sections.

Since the horizontal dimensions of the sections are small compared to the wavelength, the quantities \dot{w}_n and \ddot{w}_n may be taken as the vertical particle velocity and acceleration in the center.

$$\dot{w}_n \approx \frac{\partial \eta_z}{\partial t} = - \frac{A\omega \sinh k(d + z - D_n)}{\sinh kd} \sin(\omega t + \epsilon) \quad (4)$$

$z = Z - D_n$

$$\ddot{w}_n \approx \frac{\partial^2 \eta_z}{\partial t^2} = \frac{A\omega^2 \sinh k(d + z - D_n)}{\sinh kd} \cos(\omega t + \epsilon) \quad (5)$$

$z = Z - D_n$

where

D_n = distance between the still water level and the average level of the bottom of the section.

Since the displacements (Z) are generally small, compared to $(d - D_n)$, Equations (4) and (5) reduce to

$$\dot{w}_n \approx - A\omega \frac{\sinh k(d - D_n)}{\sinh kd} \sin(\omega t + \epsilon) \quad (4a)$$

$$\ddot{w}_n \approx A\omega^2 \frac{\sinh k(d - D_n)}{\sinh kd} \cos(\omega t + \epsilon) \quad (5a)$$

For the vehicle as a whole, the linearized equations of vertical motion become

$$\sum_{n=1}^{n=4} F_{z_n}(p) + S_r = M_t \ddot{Z} + M''_{z_t} \ddot{\bar{Z}} - \sum_{n=1}^{n=4} M''_{z_n} W_n - \sum_{n=1}^{n=4} N_{z_n} W_n + N_{z_t} \dot{\bar{Z}} + K_t S \quad (6)$$

where

$M_t = \sum M_n$ = total mass of the vehicle

M''_{z_t} = total added mass of the vehicle

N_{z_n} = linearized damping coefficient of the n^{th} section

$N_{z_t} = \sum N_{z_n}$

S_r = external vertical restraining force of the vehicle

The total vertical upward force on the horizontal planes of the section are calculated from the pressures at the depths of these planes. The pressure (p) at a depth z is

$$p = p_a - \rho g z + \rho g A \frac{\cosh k(z+d)}{\cosh kd} \cos(\omega t + \epsilon) \quad (7)$$

where p_a = atmospheric pressure

As the planes are small compared to the wavelength, the pressures on them are assumed to be uniform and equal to the pressure in the center of a plane.

The force $\sum_{n=1}^{n=4} F_{z_n}(p)$ producing the heaving motion of the vehicle (Fig. 3) is

$$\sum_{n=1}^{n=4} F_{z_n}(p) = \sum_{n=1}^{n=4} (p - p_a)_z = z - D_n (A_n - A_{n+1}) \quad (8)$$

where A_n = horizontal cross-sectional area of the n^{th} section

Using Eqs. (7) and (8), the force $\sum_{n=1}^{n=4} F_{z_n} (-)$ is determined as

$$\sum_{n=1}^{n=4} F_{z_n} (p) = \rho g \vec{v}_t - \rho g A_1 Z + \frac{\rho g A}{\cosh kd} \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n + Z) \quad (9)$$

where \vec{v}_t = total displacement of the vehicle.

If the vehicle is not restrained ($S_r = 0$), then

$$\rho g \vec{v}_t = M_t \ddot{Z} \quad (10)$$

Since the displacements (Z) are small, it may be assumed that

$$\cosh k(d - D_n + Z) = \cosh k(d - D_n) \quad (11)$$

The use of Eqs. (9), (10), and (11) in Eq. (6) results in

$$(M_t + M''_{z_t}) \ddot{Z} + N_{z_t} \dot{Z} + \rho g A_1 Z = \sum_{n=1}^{n=4} M''_{z_n} W_n + \sum_{n=1}^{n=4} N_{z_n} W_n \\ + \frac{\rho g A \cos(\omega t + \epsilon)}{\cosh kd} \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n) \quad (12)$$

Equation (12) is in the form of the equation of motion of a linear damped mass-spring system. The mass in this system, used as an analogue (Fig. 4), consists of the mass of the vehicle M_t and the so-called added mass M''_{z_t} , which conveniently may be imagined as an amount of water that is moving together with the vehicle. The damping force is the system is the linearized drag of the vehicle when moving vertically in water without the disturbance of the waves. The restoring force of the system is the restoring force due to buoyancy, which is equal to the weight of the displaced water for any given vertical displacement of the vehicle from its free-floating condition.

The excitation of this analogous system is represented by the term on the right side of Eq. (12). The last term on the right side represents the excitation by the pressure variations of the wave on the horizontal planes of the vehicle; the second term on the right side represents the excitation by the flow of the water around the body in the vertical direction (drag). The first term on the right side is difficult to visualize. In essence, it represents an inertia excitation, which is the added mass times the acceleration of the water particles at the horizontal planes.

All three terms are periodic forces with the frequency of the wave. It will be noted from Eqs. (4), (5), and (12) that these periodic forces are not all in phase with the wave. The magnitude of the total periodic force and its phase angle relative to the wave can be calculated by a vectorial summation of the three components of the excitation. Consequently, the excitation may be expressed as

$$\sum_{n=1}^{n=4} M''_{z_n} \dot{w}_n + \sum_{n=1}^{n=4} N_{z_n} w_n + \frac{o g A \cos(\omega t + \epsilon)}{\cosh kd} \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n)$$

$$= F_{ex} \cos (\omega t + \epsilon + \alpha) \quad (13)$$

where F_{ex} = periodic excitation of the analogous system

α = phase angle

It will be noted that the magnitude of the pressure term in Equation (12) and (13) is dependent on the wave frequencies as it constitutes the sum of the pressures on four surfaces. Because of decreasing pressures with increasing depths, the total force can be zero for a particular wave frequency.

If no wave action is present, the right side of Eq. (12) becomes zero.

$$(M_t + M''_{z_t}) \ddot{Z} + N''_{z_t} \ddot{Z} + \sigma g A_1 Z = 0 \quad (14)$$

Equation (14) presents the decaying motions in heave of the vehicle in still water after the vehicle has been displaced from its equilibrium position.

The motions of the vehicle in uniform waves with a particular height and period can be calculated with Eq. (12) if the dimensions and the added mass and damping terms are known.

For bodies with a simple geometry, e.g., spheres or cylinders, values of the added mass can be calculated by theory.^(12,13) The added mass of more complicated bodies is generally determined from model experiments in still water by use of Eq. (14), or--in the case of ships--is calculated from graphs which are based on model experiments. Since the added mass M''_{z_t} can be obtained from such a simple experiment, no attempt has been made to derive an analytical value.

The added-mass terms M''_{z_n} may possibly be estimated in proportion to the areas at the transition of the sections from M''_{z_t} .

No damping coefficients were found in the available literature for bodies with a shape similar to the vehicle being considered. By means of a simple test and use of Eq. (14), a value of the linearized damping coefficient N''_{z_t} due to viscous and wave-generating effects could be obtained for the vehicle model when moving in its natural frequency. This value, however, may deviate considerably from the prototype values, due to differences in Reynolds numbers.

However, since Eq. (12) represents a damped linear system, the value of damping coefficients is materially unimportant for the calculation of the response for motions with a frequency above twice the natural frequency of the system.

Studies by Keuligan and Carpenter for cylinders and plates⁽¹⁶⁾ and Harleman and Shapiro for spheres⁽¹⁷⁾ indicate that for large objects the drag forces are small compared with the inertia forces, and in many cases may be disregarded.

Consequently, for frequencies above twice the natural frequency, Eq. (12) is reduced to

$$(M_t + M''_{z_t}) \ddot{Z} + \rho g A_1 Z = \sum_{n=1}^{n=4} M''_{z_n} \dot{W} + \frac{\rho g A \cos(\omega t + \epsilon)}{\cosh kd} \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n) \quad (15)$$

The steady state solution of Eq. (15) is

$$Z = \frac{1}{1 - \left(\frac{\omega}{\omega_0}\right)^2} \cdot \frac{1}{\rho g A_1} \sum_{n=1}^{n=4} M''_{z_n} \dot{W} + \frac{\rho g A \cos(\omega t + \epsilon)}{\cosh kd} \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n) \quad (16)$$

where

$$\omega_0 = \frac{\rho g A_1}{(M_t + M''_{z_t})}$$

Pitch and Surge

In the following analyses it is assumed that the vehicle consists of a series of coupled cylinders, each with a different length and diameter. Next, it is assumed that the horizontal displacement of each cross section of a cylinder is equal to the displacement of its center.

Using the same coordinate system as is used for the analysis of the heaving motion, and considering the mass center of the vehicle, the equations of the surge X and pitch ϕ are expressed by

$$\sum_{n=1}^{n=4} F_{x_n}(p) = M_t \ddot{X} - \sum_{n=1}^{n=4} M''_{x_n} (\dot{U}_n - \ddot{X} - r_n \ddot{\phi}) + \\ \frac{1}{2} \sum_{n=1}^{n=4} C_d \rho L_n 2R_n (\dot{U}_n - \dot{X} - r_n \dot{\phi})^2 \quad (17)$$

$$\sum_{n=1}^{n=4} F_{x_n}(p) r_n = I \ddot{\phi} - \sum_{n=1}^{n=4} M''_{x_n} r_n (\dot{U}_n - \dot{X} - r_n \dot{\phi}) + \\ \frac{1}{2} \sum_{n=1}^{n=4} C_d \rho L_n 2R_n r_n (\dot{U}_n - \dot{X} - r_n \dot{\phi})^2 \quad (18)$$

where

X = horizontal displacement of the mass center

\dot{X} = horizontal velocity of the mass center

\ddot{X} = horizontal acceleration of the mass center

I = mass inertia moment of the vehicle around its mass center

r_n = distance between the mass center of a cylinder and the mass center of the vehicle

ϕ = angular displacement of the long axis of the vehicle

$\dot{\phi}$ = angular velocity of the long axis of the vehicle

$\ddot{\phi}$ = angular acceleration of the long axis of the vehicle

m_{ϕ} = metacenter height

n = cylinder number

M''_{x_n} = added mass of a cylinder moving horizontally

$F_{x_n}(p)$ = total horizontal force from the wave pressure on a cylinder

M_{x_n} = mass of a cylinder moving horizontally

L_n = length of a cylinder

R_n = radius of a cylinder

U = mean horizontal velocity of the water mass displaced by a cylinder

\dot{U} = mean horizontal acceleration of the water mass displaced by a cylinder

The mean velocity U_n and the mean acceleration \dot{U}_n are defined as

$$U_n = \frac{1}{\pi R_n^2} \int_{-R_n}^{R_n} \int_{-\sqrt{R_n^2 - y^2}}^{\sqrt{R_n^2 - y^2}} z = D_{n-1} \frac{\partial \xi_z}{\partial t} \cdot dz dx dy \quad (19)$$

$$\dot{U}_n = \frac{1}{\pi R_n^2} \int_{-R_n}^{R_n} \int_{-\sqrt{R_n^2 - y^2}}^{\sqrt{R_n^2 - y^2}} z = D_n \frac{\partial^2 \xi_z}{\partial t^2} \cdot dz dx dy$$

$$\dot{U}_n = \frac{1}{\pi R_n^2} \int_{-R_n}^{R_n} \int_{-\sqrt{R_n^2 - y^2}}^{\sqrt{R_n^2 - y^2}} z = D_{n-1} \frac{\partial^2 \xi_z}{\partial t^2} \cdot dz dx dy \quad (20)$$

$$\dot{U}_n = \frac{1}{\pi R_n^2} \int_{-R_n}^{R_n} \int_{-\sqrt{R_n^2 - y^2}}^{\sqrt{R_n^2 - y^2}} z = D_n$$

For one cylinder, the total wave pressure may be written

$$F_{x_n}(p) = \int_{-R_n}^{R_n} \left[\int_{D_n}^{D_{n-1}} (p - p_a) dz \right] dy \quad x = -\sqrt{R_n^2 - y^2}$$

$$= \int_{D_n}^{D_{n-1}} (p - p_a) dz dy \quad x = \sqrt{R^2 - y^2} \quad (21)$$

where the wave pressure p is given by

$$p = p_a - \rho g z + \rho g A \frac{\cosh k(z+d)}{\cosh kd} \cos(\omega t + \epsilon) \quad (22)$$

Evaluation of the integrals in Eq. (21) by use of Eqs. (20) and (22) gives with some approximation

$$\sum_{n=1}^{n=4} F_{x_n}(p) = \sum_{n=1}^{n=4} \rho V_n \dot{U}_n \quad (23)$$

$$\sum_{n=1}^{n=4} F_{x_n}(p) f_n = \sum_{n=1}^{n=4} \rho V_n \dot{U}_n - M g z_d \phi \quad (24)$$

where

V_n = volume of the section

By use of Eq. (23) in Eq. (17) and Eq. (24) in Eq. (18), and by linearizing the damping terms, the equations of motions may be expressed by

$$(M_t + M''_{x_t}) \ddot{X} + N_{x_t} \dot{X} + \sum_{n=1}^{n=4} M''_{x_n} \dot{z}_n \phi + \sum_{n=1}^{n=4} N_{x_n} f_n \dot{\phi} \\ - \sum_{n=1}^{n=4} (\rho V_n + M''_{x_n}) \dot{z}_n + \sum_{n=1}^{n=4} N_{x_n} \dot{f}_n \quad (25)$$

$$(I + I'') \ddot{\phi} + \sum_{n=1}^{n=4} N_{x_n} f_n^2 \dot{\phi} + M g m_\phi \dot{\phi} + \sum_{n=1}^{n=4} M''_{x_n} f_n \ddot{x} + \sum_{n=1}^{n=4} B_{x_n} f_n \dot{x} \\ = \sum_{n=1}^{n=4} (\rho \nabla_n + M''_{x_n}) f_n \dot{U}_n + \sum_{n=1}^{n=4} N_{x_n} f_n U_n \quad (26)$$

where N_{x_n} = damping term of the n^{th} section in surge

M''_{x_t} = total added mass in surge

I'' = added-mass inertia moment

It will be noted that Eq. (25) (pitch equation) represents a damped mass-spring system. The excitation of this system is coupled with the movement in surge.

Following the theoretical derivations used for the heave and again disregarding the drag excitation, the equations of motion for frequencies larger than twice the natural frequency in pitch are

$$(M_t + M''_{x_t}) \ddot{x} + \sum_{n=1}^{n=4} M'_{x_n} f_n \ddot{\phi} = \sum_{n=1}^{n=4} (\rho \nabla_n + M''_{x_n}) \dot{U}_n \quad (27)$$

$$(I - I'') \ddot{\phi} + M g m_\phi \dot{\phi} + M''_{x_n} f_n \ddot{x} = \sum_{n=1}^{n=4} (\rho \nabla_n - M''_{x_n}) f_n \dot{U}_n \quad (28)$$

The solution for pitch is

$$\ddot{\phi} = \frac{\sum_{n=1}^{n=4} (\rho \nabla_n + M''_{x_n}) f_n \dot{U}_n - \frac{\sum_{n=1}^{n=4} M''_{x_n} f_n}{(M_t + M''_{x_t})} \sum_{n=1}^{n=4} (\rho \nabla_n + M''_{x_n}) \dot{U}_n}{\sum_{n=1}^{n=4} M''_{x_n} f_n^2} \quad (29)$$

$$(\frac{2}{\omega_n^2} - \frac{2}{\omega^2}) I + I'' - \frac{\sum_{n=1}^{n=4} M''_{x_n} f_n}{(M_t + M''_{x_t})}$$

where ω = natural frequency

The added mass for a cylinder accelerated in a direction normal to its axis is equal to the water mass displaced by it.^(12,13) Since for small angular movements each section of the vehicle is accelerated in a direction which is approximately in the direction of its axis, it may be assumed that $I = I''$.

For vehicles with a relatively even distribution of mass, the static moment of the added mass around the center of gravity of the vehicle is

$$\sum M_{x_n}'' f_n = \sum \rho \nabla_n f_n = 0 \quad (30)$$

In this particular case, the solution for pitch (Eq. (29)) reduces to

$$\phi = \frac{\sum (\rho \nabla_n + M_{x_n}'') f_n \dot{U}_n}{(\omega_n^2 - \omega^2) (I + I'')} = \frac{\sum \rho \nabla_n f_n \dot{U}_n}{(\omega_n^2 - \omega^2) I} \quad (31)$$

THEORY OF MOVEMENTS INDUCED BY A SEAWAY

In order to describe the movements of the vehicle in a seaway, terms of practical significance to the user have to be selected. At present, descriptions of complex movements are generally given in statistical values of the movement; to conform with this practice, this method of expression has been used herein.

The calculation method of significant values of responses of systems subjected to random or complex excitations was developed by electrical engineers for use in random noise analyses⁽¹⁸⁾ and was more recently applied to the calculation of ship movements in a seaway.⁽⁴⁾

The basic requirement for the applicability of the method is that the response of the considered system be linear to the excitation. If the method is applied to the analogous linear system of the vehicle, as described earlier in connection with Fig. 4, then the responses of the vehicle to a sum of uniform wave trains -- which are assumed to be moving in one direction -- are equal to the sum of the vehicle responses of the individual components.

Since the responses of the vehicle in uniform wave trains can be determined by theory or by model experiments, the complex movements excited by waves with a certain spectrum can be obtained by means of the following procedure, which is illustrated in Fig. 5.

For all frequencies of the considered wave spectrum, the value of the mean square of the wave amplitudes per unit of wave frequency -- called spectral density of the waves -- is multiplied by the corresponding square of the ratio of response to wave amplitude, and the results are plotted as

the spectral density of the response. In mathematical terms, the procedure is generally written as follows:

$$S_r(\omega) = S_w(\omega) \cdot [T(\omega)]^2$$

where $S_r(\omega)$ = spectral density at frequency ω of the response

$S_w(\omega)$ = spectral density at frequency ω of the waves

$[T(\omega)]^2$ = square of the ratio of response to wave amplitude at frequency ω of the wave environment

Once the spectrum of the movement in study has been obtained, an integration of the spectrum results in a single number E , which represents the energy of the movement. If the spectrum is relatively narrow, the average amplitudes of the movement can be expressed in terms of the energy E , as shown in Table I.

Table 1
RESPONSE-AMPLITUDE (A) AND RESPONSE-ENERGY (E) RELATIONS
(Based on Ref. 3, after Ref. 1)

Average-Response-Amplitude Data

Average amplitude of all responses	$A_{av}^{resp} = 0.886 \sqrt{E}$
Average amplitude of the 1/3 highest	$A_{1/3}^{resp} = 1.416 \sqrt{E}$
Average amplitude of the 1/10 highest	$A_{1/10}^{resp} = 1.800 \sqrt{E}$

Greatest-Response-Amplitude Probability Data

Number of cycles (N) Nine times out of ten, the highest amplitude of the response out of a series of N cycles is between

20	$1.40 \sqrt{E}$	and	$2.44 \sqrt{E}$
100	$1.68 \sqrt{E}$		$2.75 \sqrt{E}$
500	$2.26 \sqrt{E}$		$3.03 \sqrt{E}$
1000	$2.41 \sqrt{E}$		$3.14 \sqrt{E}$

Note: The total deviations of the response from minimum to maximum are twice the values of the amplitudes in the table

TEST FACILITIES AND PROCEDURES

The experimental investigations were performed in a water-wave tank 100 ft long, 4 ft high, and 2 ft wide.

At one end of the tank, waves could be generated by a bulkhead-type wave generator. A gravel beach with a 1-10 slope at the other end of the tank was installed to absorb the generated waves and to reduce the generation of standing waves.

The vehicle models used in the experimental investigation were wooden models, $17\frac{1}{2}$ in. long and $1\frac{1}{4}$ in. in diameter, representing a 1-120 scale model of a million-pound solid-propellant rocket vehicle, 175 ft long and $12\frac{1}{2}$ ft in diameter.

The movements of one of the models were sensed by use of two displacement gages (Fig. 6). Each gage consisted of a thin wire soldered to two very thin brass plates, one located vertically, the other horizontally. Strain gages were mounted on these plates. Movements of the ends of the wires (arms) of the displacement gages induced strains in the strain gages, which were amplified by carrier amplifiers and recorded simultaneously with the output of a resistance-type water-level recorder in a high-speed oscillograph.

It was considered that the displacement gages did not restrain the model in its orbital movements, since the forces necessary to displace the end of the gages were insignificant (approximately 50 mg per inch deviation).

The movements of the model during experimental test runs were recorded from the moment the first wave reached the model until obvious irregularities due to standing waves were generated in the tank.

To obtain information about the characteristics of the vehicle in free heave and in free pitch, the model was displaced vertically and horizontally and suddenly released after each displacement. The movements were recorded until they had decayed to insignificant levels.

To check on the results obtained with the displacement gages, photographic observations were made with a second model. This model was equipped with two small light bulbs, one at the head and one at the tail. Power to these bulbs was supplied by means of two wires 0.005 in. diameter. (These wires were considered not to restrain the motions of the model.) By means of a mechanically operated switch, the light bulbs flashed on for 0.02 sec every 0.15 sec. By means of simple electrical circuits, the flashes and the time of exposure of the photographic film in the camera were recorded which was located near the model. The scale on the photographs was obtained by use of white wire grids placed at both sides of the tank (Fig. 1).

III. RESULTS

UNIFORM WAVE TRAINS

Experimental Results

Figures 7 and 8 present all experimental results as functions of the wave frequency. (Theoretical results, which will be treated later in this report, are plotted for comparison.) Figure 7 shows the results of all data analyses for heave. The data are presented as plots of the ratios of double heave amplitude to wave height (H_h/H), which equals the ratio of heave amplitude to wave amplitude (A_h/A) as a function of the wave frequency and period. It will be noted that the ratios increase with a decrease in frequency.

Figure 8 shows the results of all data analyses of the maximum angular deviations (pitch) from the vertical. The data are presented as plots of the ratio of the maximum angular deviation in minutes of arc to the wave height in tenths of an inch as a function of the wave frequency. The corresponding prototype frequencies and periods are indicated at the top of the graph. The prototype ratios are expressed as the maximum angular deviation in minutes of arc to the wave height per foot.

Figure 9 presents the results of a photographic observation. The shutter of the camera was opened for approximately one second. The time of the opening was recorded by the oscillograph, together with the time of the light flashes on the head and tail of the model. The output of the wave recorder was also recorded. The positions of the head and tail of the model can be identified in the photograph, which is reproduced in Fig. 9. In order to show the positions of the vehicle in more detail, the positions of the head and tail are enlarged ten times, and lines are drawn representing the movements of the long axis, enlarged ten times.

No measurements were made of sway, yaw, and roll. Visual observations of the movements of the vehicle in uniform waves in the wave tank indicated that these motions were negligible.

Within the range of the wave heights used in the experimental investigations equivalent to 3- to 7-ft-high waves for the prototype, the responses of the model in heave and pitch were significantly linear with the wave height. For example, it is estimated that the experimentally obtained ratio of heave to wave height for a wave height of 6 ft in prototype is approximately 10 per cent smaller than the ratio for a wave height of 3 ft in the prototype.

Theoretical Results

The added mass for these calculations was determined by use of the results of a modal test in still water. In this test, the model was displaced vertically and released. The following decaying motions were measured and, by use of Eq. (15), the added mass calculated.

The results of the calculations for the heaving motion were obtained by use of Eq. (16a). The experimental results presented in Fig. 7 pertain to vehicle-model movements in a water depth of 2.68 ft (prototype 320 ft). Since the particle motions of waves with a length larger than twice the water depth are affected by the bottom, the response in deep water ($d > 1/2\lambda$) will deviate from the response at a depth of 2.68 ft (prototype 320 ft).

The results of the theoretical study of the heave are in good agreement with the results of the tests made in the wave tank. The maximum deviation of the theoretical values from the average values obtained by experiments is approximately 20 per cent.

The results of the calculation for pitch for the model were made by use of Eq. (29) and plotted in Fig. 8. The calculations were made by using the experimentally determined natural frequency in pitch. Again the theoretical results are in fair agreement with the experimental results.

Seaway - Theoretical Results

As an illustration of the application of spectrum calculations, the results for heave and pitch of the prototype vehicle acted upon by nonuniform waves in deep water are presented in Table II. A fully arisen sea induced by wind speeds of 20 kn was selected as a typical seaway. This wave environment represents conditions for a Sea State 4 (Hydrographic Office Scale) without swells.

The calculations were made by use of the results of the theoretical study of heave of the vehicle in uniform deep-water waves (Fig. 7) and the results of the experimental study for pitch (Fig. 8). Figure 10 presents the calculations according to the pattern presented in Fig. 6 for the vehicle movement in heave.

Table 2

RESULTS OF CALCULATIONS FOR HEAVE AND PITCH
IN A FULLY ARISEN SEA

Characteristic Statistical Value	Fully Arisen Sea with Wind Speed of 20 kn (Sea State 4)	
	Heave (double amplitude, ft)	Pitch (maximum devia- tion from the vertical, min of arc)
Average	0.8	50
Average 1/3 highest (significant)	1.3	80
Average 1/10 highest	1.7	100
In series of 1000 cycles, 9 times out of 10 the highest values is between	2.2 - 2.9	130 - 170

*For a fully arisen sea with a wind speed of 20 kn, the
respective time for 1000 cycles is approximately 2 $\frac{1}{2}$ hr.

IV. DISCUSSION

It appears from Figs. 7 and 8 that the experimental results are in fairly good agreement with the theoretical results. Discrepancies may be explained by the approximations made in the derivations of the formulas used and in the accuracy of the measurements. A higher accuracy will be obtained by considering the movements of the two-dimensional (horizontal) flow around cylindrical sections of the vehicle with a very small height(d_z).⁽²⁰⁾

Since the particle motions in shallow water (prototype wave periods larger than 11 sec) are affected by the bottom, the orbits are elliptical rather than circular; consequently, the movements of the vehicle in shallow water deviate from the movement in deep water, which is illustrated by the heaving motion in Fig. 7.

Since the particle motions near the surface are larger than those at a great depth and the excitations are related to the volumes and their accompanying added masses (Eqs. (13) and (29)), a reduction of the dimensions near the surface will tend to reduce the movements.

Smaller dimensions at the water line will also generally decrease the frequency of free heave and pitch, and the chance for resonance with low-frequency components of the wave spectra will be reduced. Since long and slender vehicles, as investigated, have a low damping, waves with frequencies close to the frequency of free heave may induce excessive movements.

Consequently, swells, which are waves generated at a considerable distance from the point of observation, may induce the major heaving motion for large vehicles.

Such movements can be decreased by increasing the damping. This can be accomplished by installing horizontal plates or fins on the outer surface of the vehicle, preferably close to the tail end. In addition to an

increase in damping, such plates also will increase the added mass, and consequently a favorable lower frequency in free heave will be obtained.

The application of the derived formulas for heave and pitch is permitted for frequencies larger than twice the frequencies of free heave and pitch. Thus, in the case of the studied vehicle, spectra calculations as presented in Fig. 10 must be limited to seaways which contain no frequency components lower than twice the frequency of free heave (periods larger than 3 sec).

In the spectra calculations it is assumed that all waves are propagating in one direction. In reality, however, waves are generated in different directions, but generally a very small part of the wave energy is in waves which make a significant angle (larger than approximately 15 degrees) from the main direction.⁽¹⁹⁾ Because of the small horizontal dimensions of the vehicle, the heave will not be affected by directional effects; the pitch, however, is dependent on the directional components of the spectrum.

It seems possible to determine the movements of a vehicle in ocean waves by calculating the responses at the different directions of the spectrum and a vectorial addition of the components thus obtained. Since the spread of the main wave direction is relatively small, the spread of the responses is small and its effect can be disregarded.

V. CONCLUSIONS

The movements in heave and pitch of a large rocket vehicle drifting in a vertical attitude in uniform waves can be calculated by means of the theory presented in this paper if the periods of the waves are shorter than half the periods of free heave and pitch. The results of such calculations for uniform waves can be used for the calculation of the frequency of occurrence of certain levels of the heave and pitch response of the vehicle to a seaway with a given wave spectrum.

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(OFFICIAL NAVY PHOTOGRAPH)

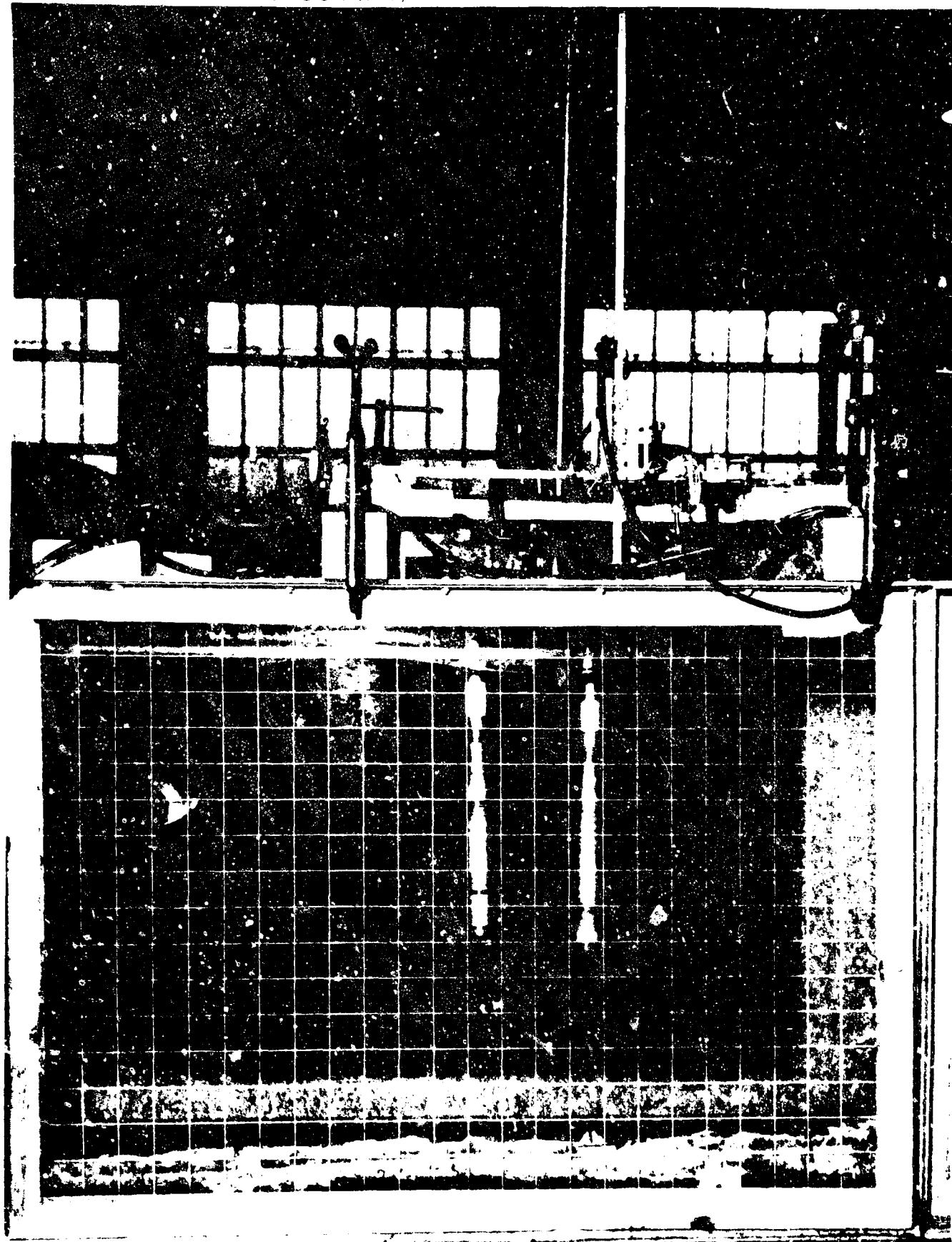


Fig. 1 — One-to-120 scale models of the rocket vehicle
in the wave tank (observation grid width is 2 in.)

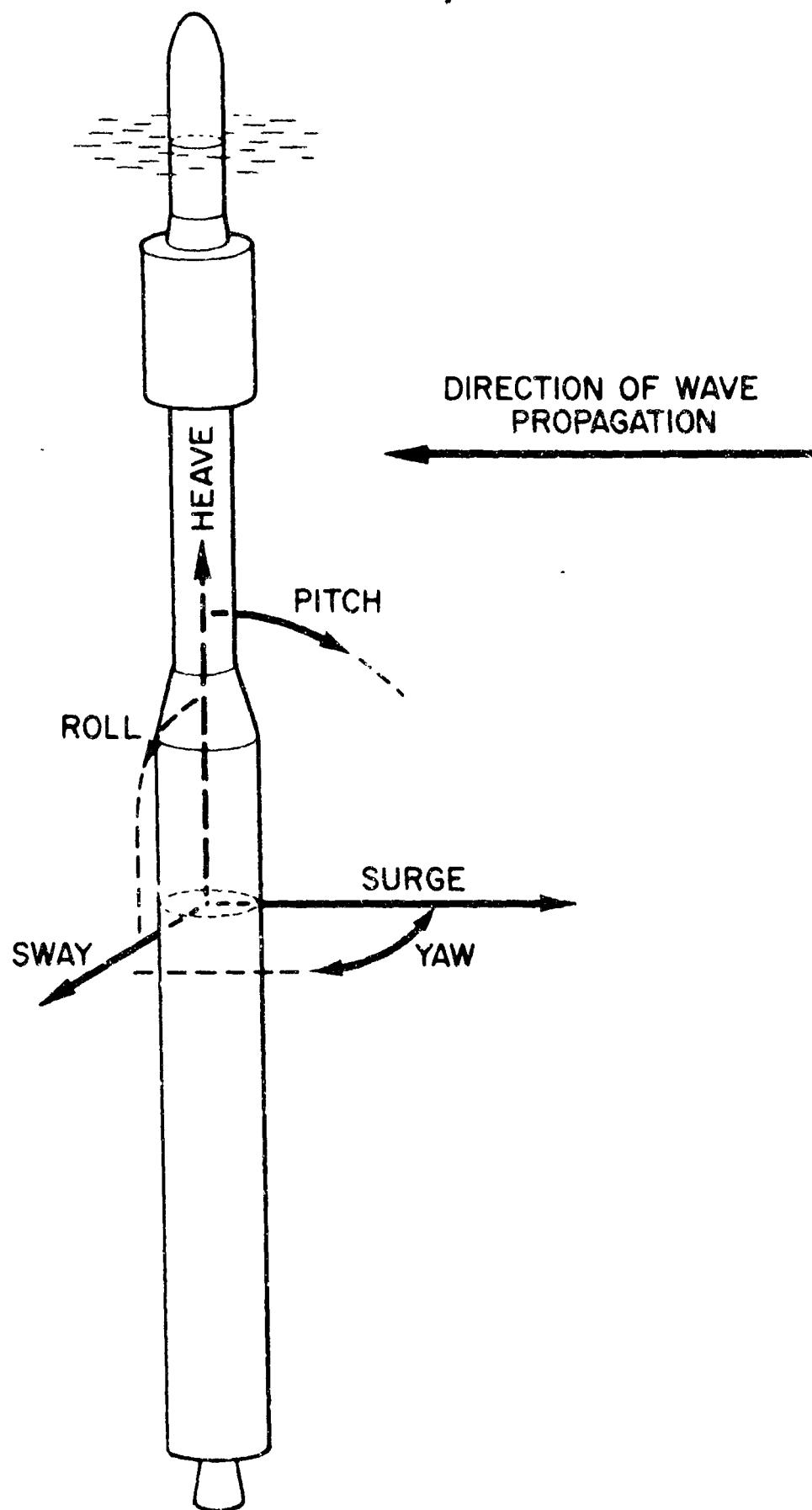


Fig. 2 — Nomenclature of the vehicle movements

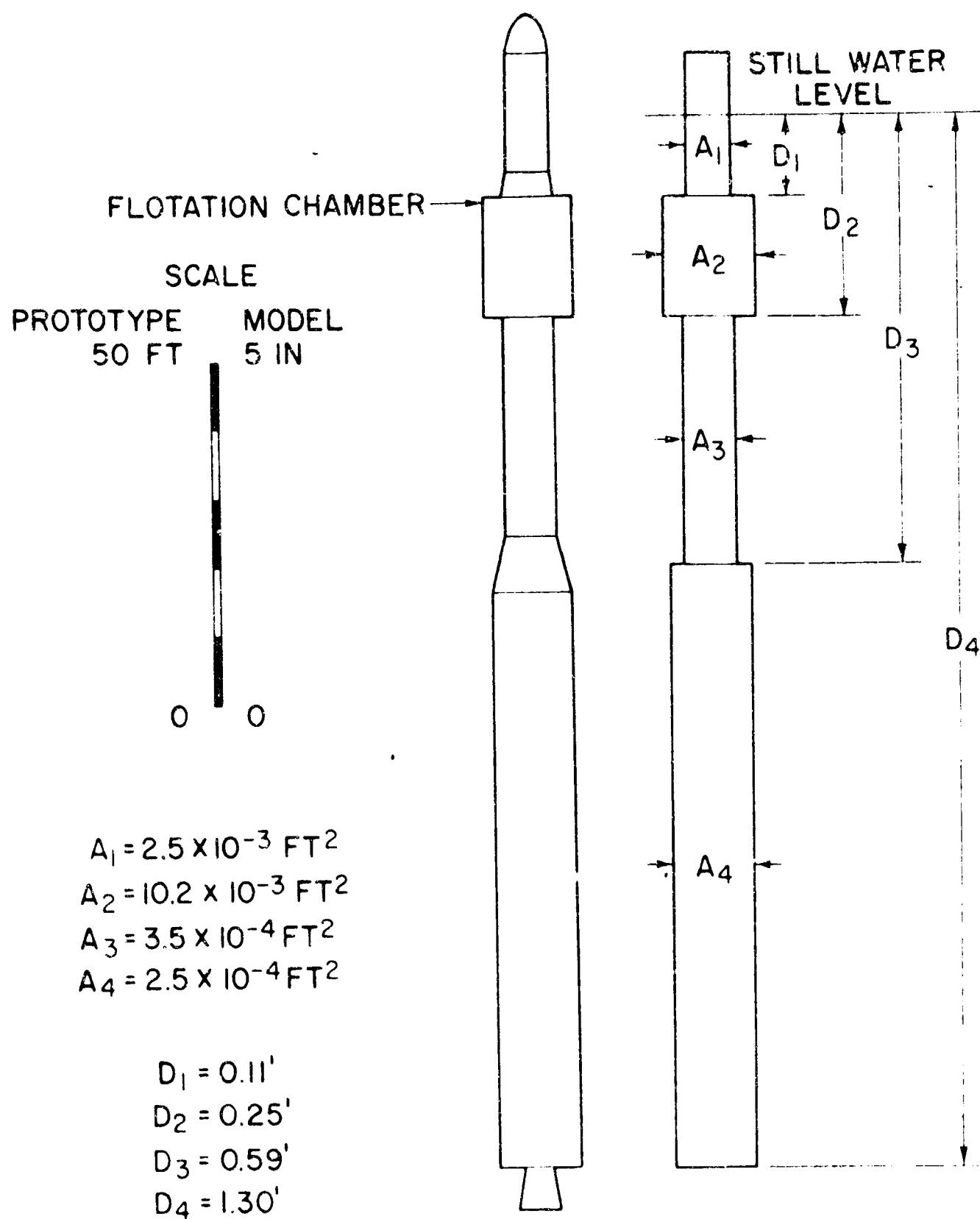
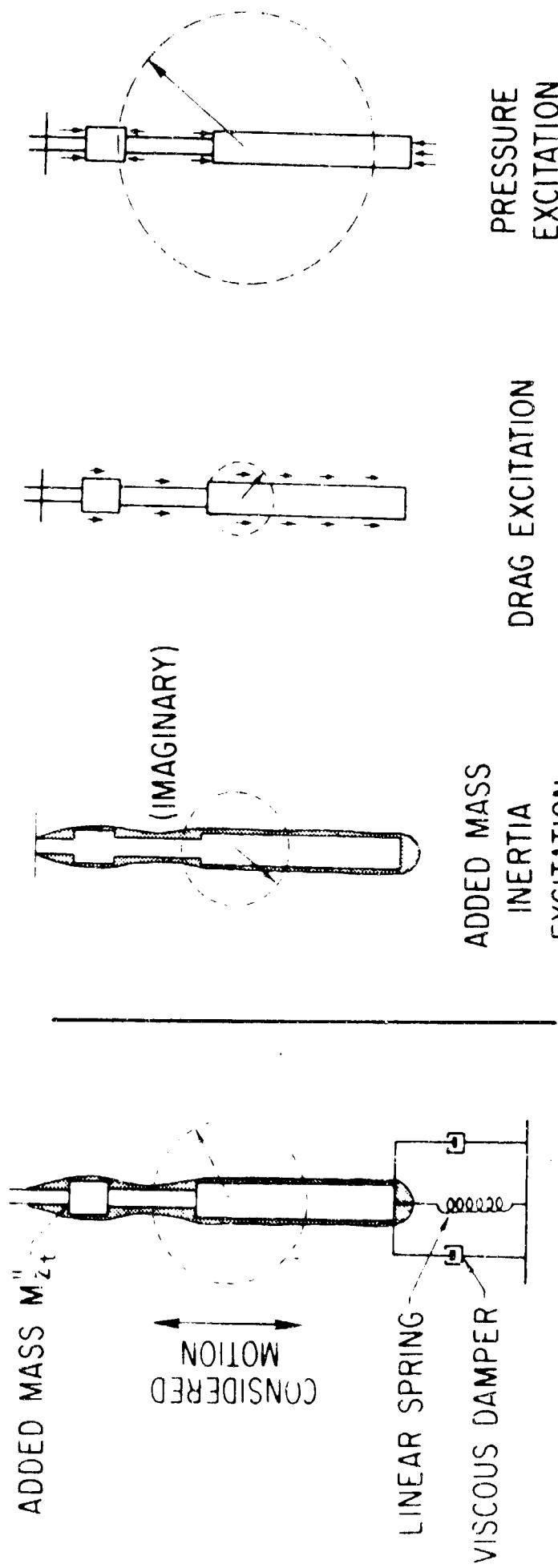


Fig. 3 — Approximation of the model dimensions
for the analyses



$$(M_t + M_{z_t}'') \ddot{Z} + -t \dot{Z} + \rho g A_1 Z = \sum_{n=1}^{n=4} M_{z_n}'' \dot{W}_n + \sum_{n=1}^{n=4} N_{z_n} W_n + \frac{\rho g A \cos(\omega t + \epsilon)}{\cosh k d} \quad \sum_{n=1}^{n=4} (A_n - A_{n+1}) \cosh k(d - D_n)$$

Fig. 4 — Mechanical analogy of the vehicle in heaving motion

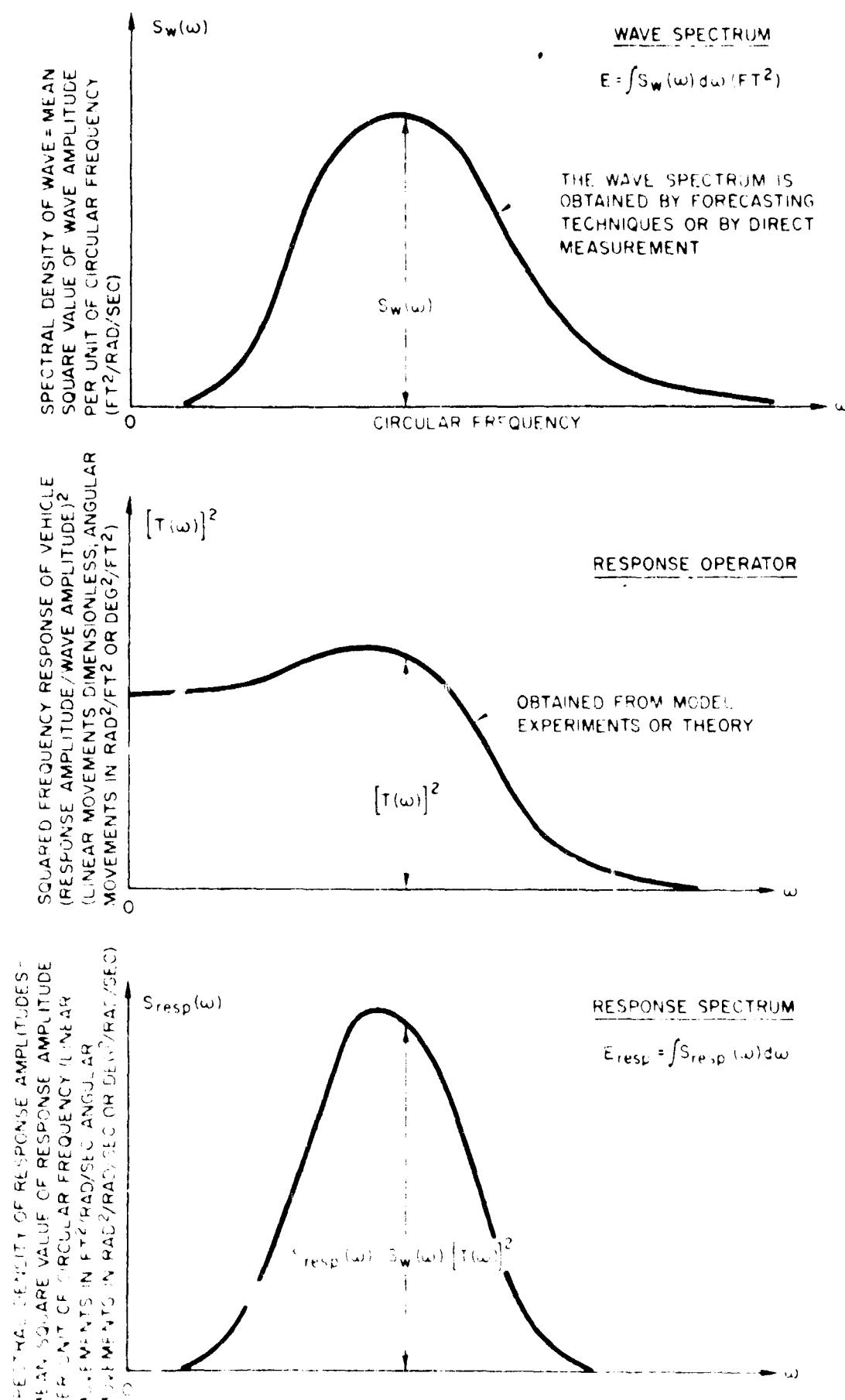


Fig. 5 -- Calculation of a vehicle - response spectrum from a given wave spectrum

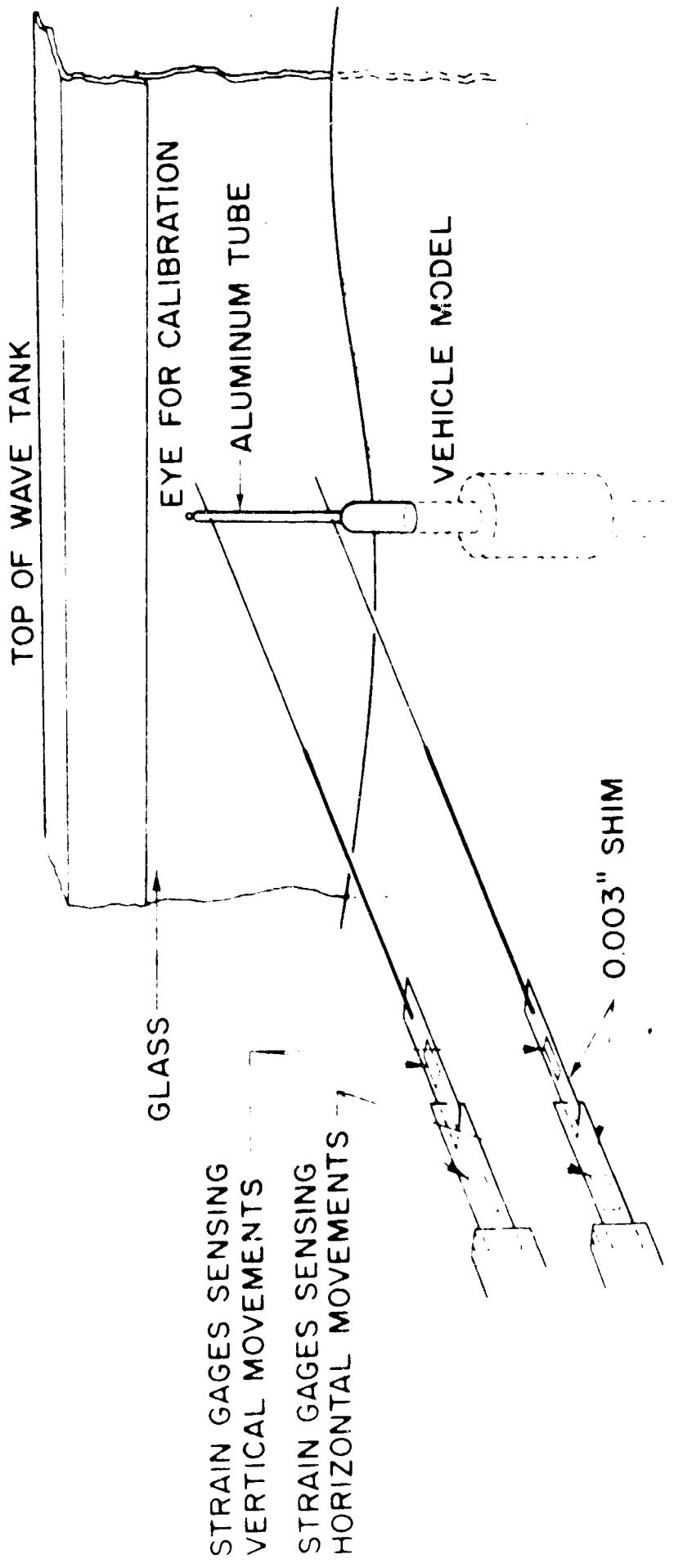


Fig. 6 — Schematic view of the displacement gages

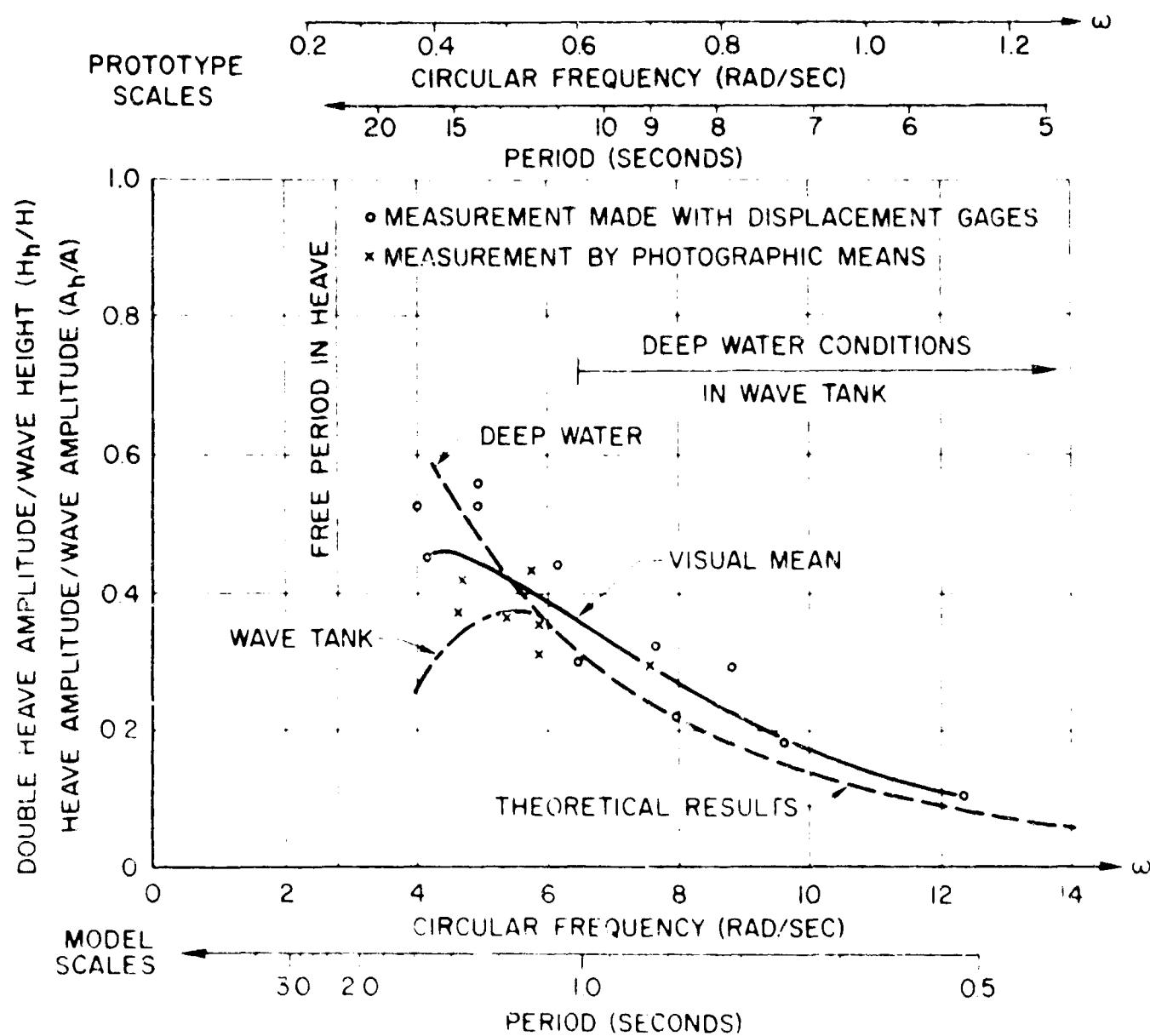


Fig. 7 — Ratio of heave to wave height as a function of the wave frequency for the model in the wave tank and the prototype in 320-ft and in deep water

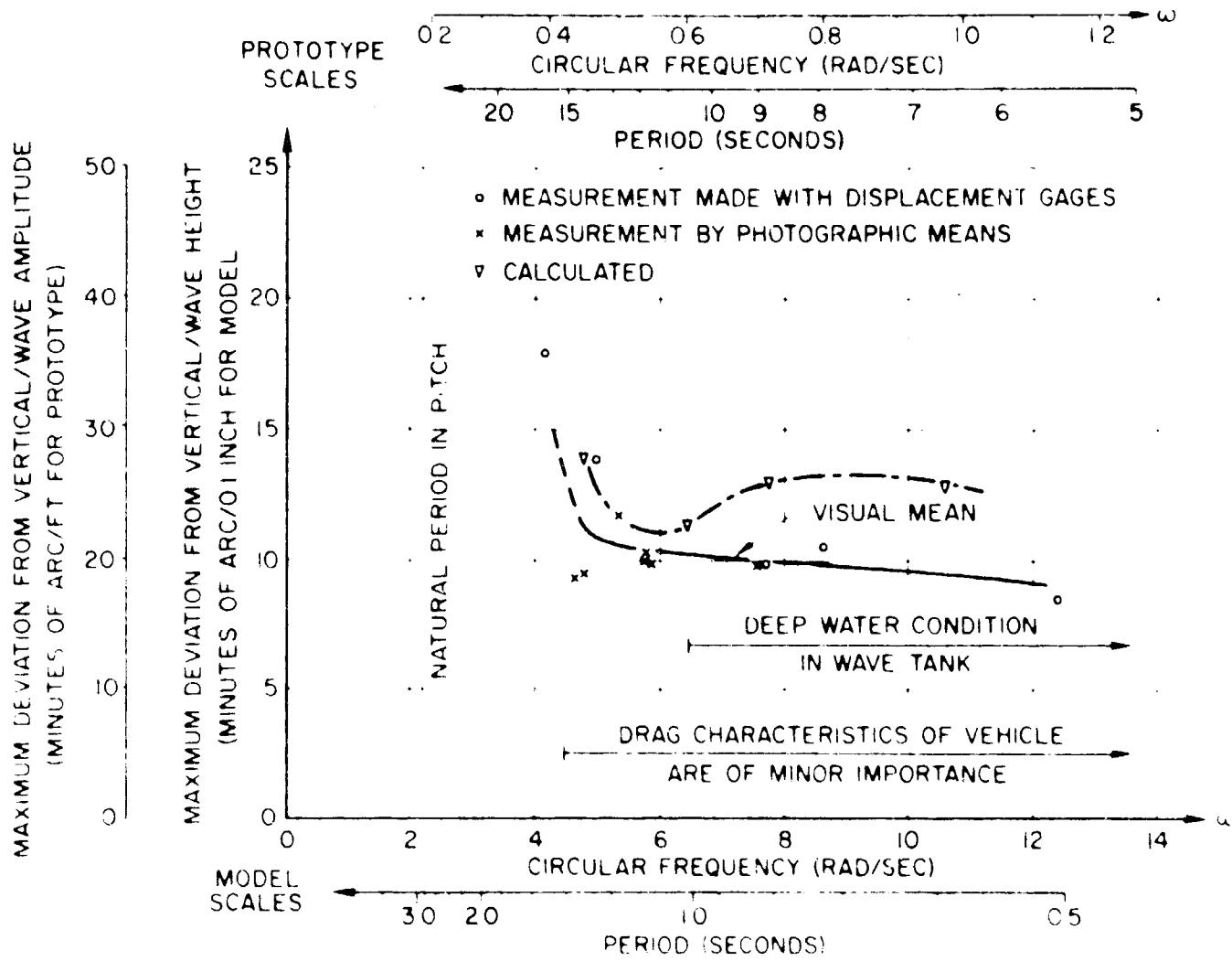


Fig 8 -- Ratio of maximum deviation from the vertical to wave height as a function of the wave frequency for the model in the wave tank and the prototype in a 320-ft water depth

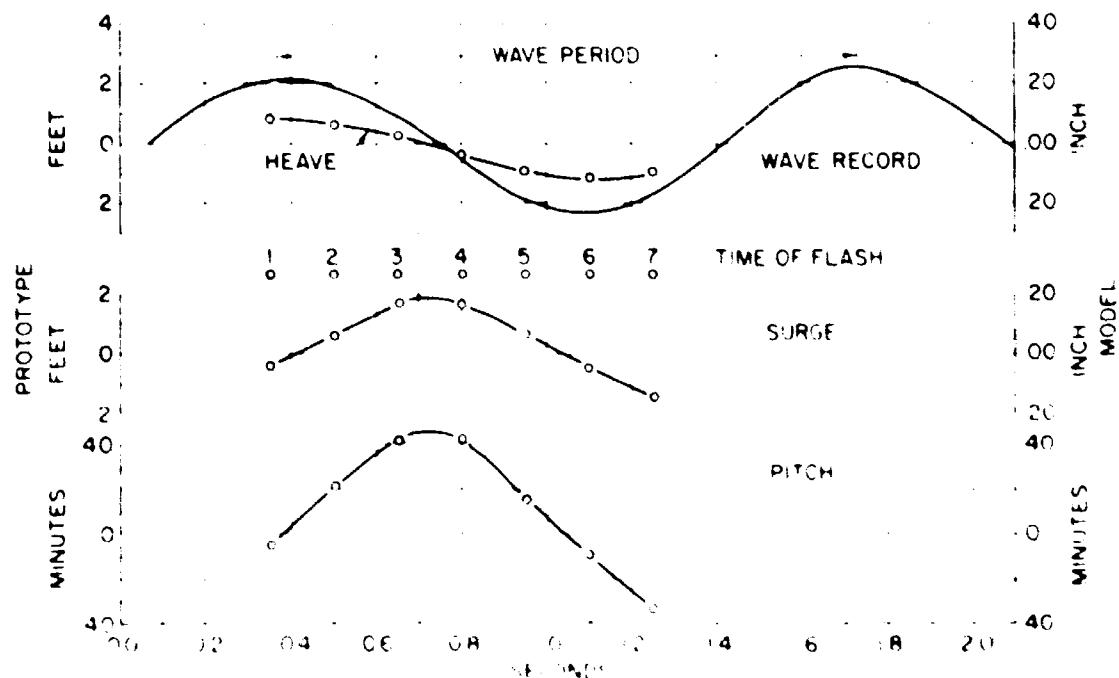
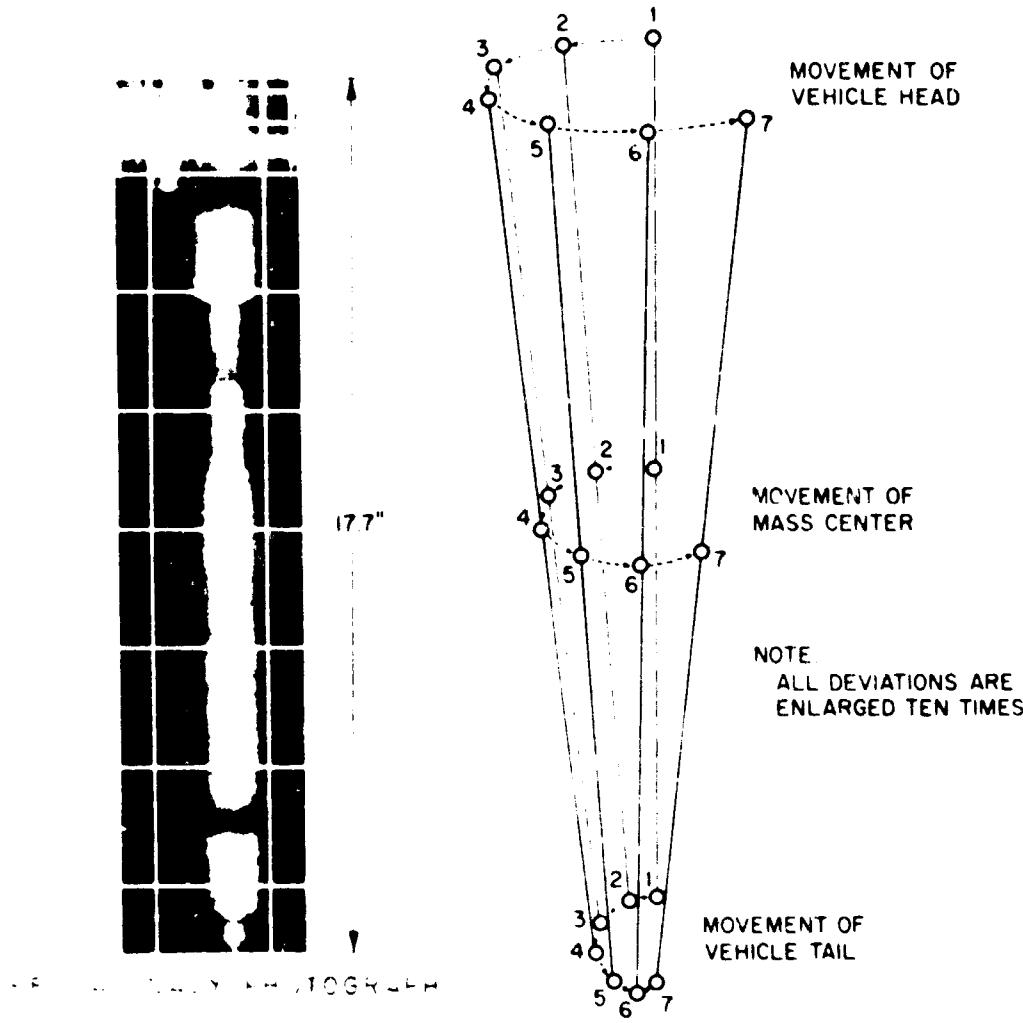


Fig 9 — Vehicle model movements (model wave height 0.90 in., period 1.34 sec, prototype wave height, 4.8 ft, period 14.7 sec)

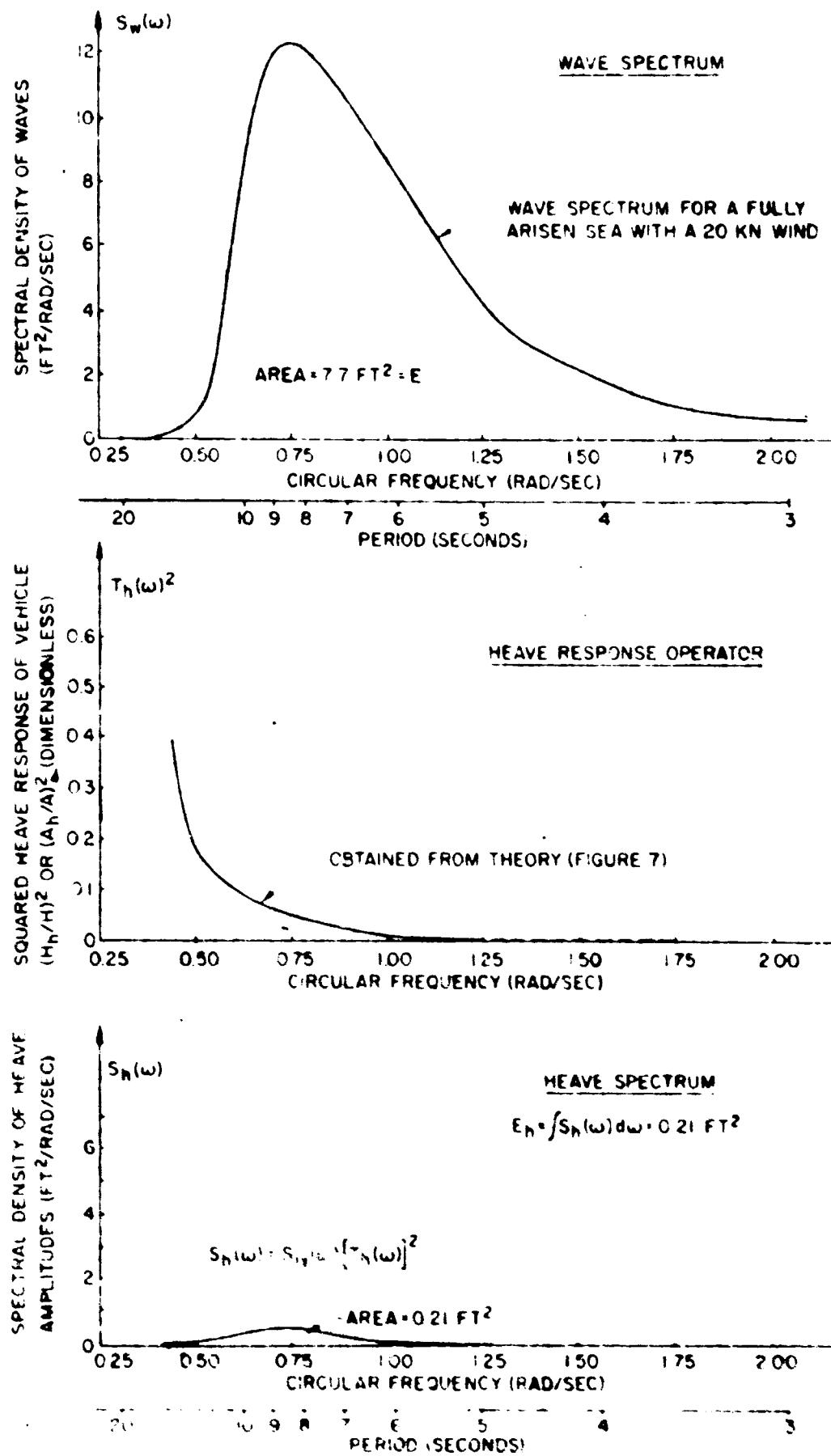


Fig 10 — Calculation of the heave spectrum in a fully arisen sea with a wind speed of 20 kn